Unit 02

ELIMINATION

Q.1 Define the following.

(i) Elimination **(Lahore Board 2009)**

By elimination of one or more than one variables from the given simultaneous equations, we get such a relation which is independent of that variable. This procedure is called elimination.

OR

The procedure to find a relation from two equations independent of a variable involved in both the equations is called elimination.

(ii) Methods of Elimination:

Following methods are used for elimination

(i) Elimination of one variable by substitution or by comparison method.

(ii) Elimination of one variable by applying formulae.

(iii) Elimination of one variable by using the method of cross multiplication.

(iii) Conditions for Elimination

(i) At least two equations are required for elimination of one variable.

(ii) Both equations should have the same variable that has to be eliminated.

(iii) Eliminant or relation shows that the solution set of both equations is not empty.

(iv) Eliminant **(Lahore Board 2010)**

Result/relation obtained by elimination is called eliminant.

EXERCISE 2.1

Q.1 Eliminate x from the following equations by comparison method.

(i) $ax - b = 0$; $cx - d = 0$

Sol: Given eqs. are

$$ax - b = 0 \quad \text{.........} (1)$$
$$cx - d = 0 \quad \text{.........} (2)$$

from (1) $x = \dfrac{b}{a}$ .... (3)

from (2) $x = \dfrac{d}{c}$ .... (4)

Comparing (3) and (4), we get

$$\frac{b}{a} = \frac{d}{c}$$
$$ad = bc$$

(ii) $2x + 3y = 5$; $x - y = 2$

Sol: Given eqs. are

$$2x + 3y = 5 \quad \text{.........} (1)$$
$$x - y = 2 \quad \text{.........} (2)$$

from (1) $2x = 5 - 3y$

or $x = \dfrac{5-3y}{2}$ .... (3)

from (2) $x = y + 2$ .... (4)

Comparing (3) and (4), we get

$$\frac{5-3y}{2} = y + 2$$
$$5 - 3y = 2y + 4$$
$$5 - 4 = 2y + 3y$$
$$1 = 5y$$
$$5y = 1$$

(iii) $xt = l$; $\frac{x}{m} = t$

Sol: Given eqs. are

$xt = l$ .... (1)

$\frac{x}{m} = t$ .... (2)

from (1)

$x = \frac{l}{t}$ .... (3)

from (2)

$x = mt$ .... (4)

Comparing (3) and (4), we get

$\frac{l}{t} = mt$

$l = mt^2$

or $mt^2 = l$

(iv) $x - pq = 0, \frac{x}{l} = m$

Sol: Given eqs. are

$x - pq = 0$ .... (1)

$\frac{x}{l} = m$ .... (2)

from (1) $x = pq$ .... (3)

from (2) $x = ml$ .... (4)

comparing (3) and (4)

$pq = ml$

Q.2 Eliminate x by substitution method.

(i) $ax - b = 0$; $cx - d = 0$

Sol: Given eqs. are

$ax - b = 0$ .... (1)

$cx - d = 0$ .... (2)

from (1) $x = \frac{b}{a}$

put in (2)

$c(\frac{b}{a}) - d = 0$

$bc - ad = 0$

or $ad = bc$

(ii) $2x + 3y = 5$; $x - y = 2$

Sol: Given eqs. are

$2x + 3y = 5$ .... (1)

$x - y = 2$ .... (2)

from (2) $x = 2 + y$

put in (1)

$2(2 + y) + 3y = 5$

$4 + 2y + 3y = 5$

$5y = 5 - 4$

$5y = 1$

(iii) $xt = l$; $\frac{x}{m} = t$

Sol: Given eqs. are

$xt = l$ .... (1)

$\frac{x}{m} = t$ .... (2)

from (1)

$x = \frac{l}{t}$

put in (2)

$\frac{l}{mt} = t$

$l = mt^2$

$mt^2 = l$

(iv) $x - pq = 0$; $\frac{x}{l} = m$

Sol: Given eqs. are

$x - pq = 0$ .... (1)

$\frac{x}{l} = m$ .... (2)

from (1) $x = pq$

put in (2)

$\frac{pq}{l} = m$

pq = lm

Q.3 Find a relation independent of x.

(i) $lx^2 - m = 0$; $px^2 - q = 0$

Sol: Given eqs. are

$lx^2 - m = 0$ .... (1)

$px^2 - q = 0$ .... (2)

from (1)

$lx^2 = m$

$x^2 = \frac{m}{l}$

put in (2)

$p(\frac{m}{l}) - q = 0$

$pm - ql = 0$

$mp = ql$

(ii) ax – b = 0 ; $cx^2 + dx + f = 0$

Sol: Given eqs. are

ax – b = 0 .... (1)

$cx^2 + dx + f = 0$ .... (2)

from (1) ax = b

x = $\frac{b}{a}$

put in (2)

$$c(\frac{b}{a})^2 + d(\frac{b}{a}) + f = 0$$

$$c(\frac{b^2}{a^2}) + \frac{db}{a} + f = 0$$

Multiplying both sides by a^2

$$a^2 \times c(\frac{b^2}{a^2}) + a^2 \times \frac{db}{a} + a^2 \times f = 0 \times a^2$$

or $cb^2 + abd + a^2 f = 0$

(iii) p(x+a) = q(b+x) ; p(c+x) = q(x+d)

Sol: Given eqs. are

p (x+a) = q (b+x) .... (1)

p (c+x) = q (x+d) ...... (2)

from (1) px + pa = qb + qx

px – qx = qb – pa

x (p-q) = qb – pa

x = $\frac{qb - pa}{p - q}$ .... (3)

from (2) pc + px = qx + qd

px – qx = qd – pc

x (p-q) = qd – pc

x = $\frac{qd - pc}{p - q}$ .... (4)

comparing (3) and (4), we get

$$\frac{qb - pa}{p - q} = \frac{qd - pc}{p - q}$$

or qb – pa = qd – pc

qb – qd = pa – pc

q (b-d) = p (a-c)

(iv) x + y = a ; $x^2 + y^2 = b^2$

Sol: Given eqs. are

x + y = a .... (1)

$x^2 + y^2 = b^2$ .... (2)

from (1) x = a – y

put in (2)

$(a - y)^2 + y^2 = b^2$

$a^2 - 2ay + y^2 + y^2 = b^2$

$2y^2 - 2ay = b^2 - a^2$

$2y(y - a) = b^2 - a^2$

(v) $lx + my = 1$; $(l + m)x^2 + a = 0$

Sol: Given eqs. are

lx + my = 1 .... (1)

$(l + m)x^2 + a = 0$ .... (2)

from (1)

$lx = 1 - my$

$x = \frac{1-my}{l}$

put in (2)

$(l+m)(\frac{1-my}{l})^2 + a = 0$

$(l+m)(\frac{1-2my+y^2m^2}{l^2}) + a = 0$

$(l+m)(1+m^2y^2-2my) + al^2 = 0$

Q.4 Eliminate t from the following equations.

(i) at = x ; 2at = y

Sol: Given eqs. are

at = x .... (1)

2at = y .... (2)

from (1)

$t = \frac{x}{a}$

put in (2)

$2a(\frac{x}{a}) = y$

2x = y

or y = 2x

(ii) $at^2 = x$; $bt^3 = y$

Sol: Given eqs. are

$at^2 = x$ .... (1)

$bt^3 = y$ .... (2)

from (1)

$t^2 = \frac{x}{a}$

Taking cube on both sides.

$(t^2)^3 = \left(\frac{x}{a}\right)^3$

$t^6 = \frac{x^3}{a^3}$ .... (3)

from (2)

$t^3 = \frac{y}{b}$

squaring both sides

$t^6 = \frac{y^2}{b^2}$ .... (4)

from (3) and (4), we get

$\frac{x^3}{a^3} = \frac{y^2}{b^2} \Rightarrow x^3b^2 = a^3y^2$

(iii) $2at^3 = x$; $4bt^4 = y$

Sol: Given eqs. are

$2at^3 = x$ .... (1)

$4bt^4 = y$ .... (2)

from (1)

$t^3 = \frac{x}{2a}$

Taking 4th power on both sides

$(t^3)^4 = \left(\frac{x}{2a}\right)^4$

$t^{12} = \frac{x^4}{16a^4}$ .... (3)

from (2)

$t^4 = \frac{y}{4b}$

Taking cube on both sides

$(t^4)^3 = (\frac{y}{4b})^3$

$t^{12} = \frac{y^3}{64b^3}$ .... (4)

comparing (3) and (4), we get

$\frac{x^4}{16a^4} = \frac{y^3}{64b^3}$

$\frac{x^4}{a^4} = \frac{y^3}{4b^3}$

$a^4y^3 = 4b^3x^4$

(iv) $x = \sqrt{2}\,t$; $y = \sqrt{6}\,t$

Sol: Given eqs. are

$x = \sqrt{2}\,t$ .... (1)

$y = \sqrt{6}\,t$ .... (2)

from (1)

$t = \dfrac{x}{\sqrt{2}}$

put in 2

$y = \sqrt{6}.\dfrac{x}{\sqrt{2}}$

$y = \sqrt{2}.\sqrt{3}.\dfrac{x}{\sqrt{2}}$

$y = \sqrt{3}x$

(v) $x - y = 2t$; $x^2 + y^2 = 3t^2$

Sol: Given eqs. are

$x - y = 2t$ .... (1)

$x^2 + y^2 = 3t^2$ .... (2)

from (1)

$t = \dfrac{x-y}{2}$

put in (2)

$x^2 + y^2 = 3(\dfrac{x-y}{2})^2$

$x^2 + y^2 = \dfrac{3}{4}(x^2 + y^2 - 2xy)$

$4x^2 + 4y^2 = 3(x^2 + y^2 - 2xy)$

$4x^2 + 4y^2 = 3x^2 + 3y^2 - 6xy$

$4x^2 + 4y^2 - 3x^2 + 3y^2 + 6xy = 0$

$x^2 + y^2 + 6xy = 0$

(vi) $at^3 - b = 0$; $ct^2 + d = 0$

Sol: Given eqs. are

$at^3 - b = 0$ .... (1)

$ct^2 + d = 0$ .... (2)

from (1)

$at^3 = b$

$t^3 = \dfrac{b}{a}$

Squaring both sides

$t^6 = \dfrac{b^2}{a^2}$ .... (3)

from (2)

$ct^2 = -d$

$t^2 = -\dfrac{d}{c}$

Taking cube of both sides .... (4)

$t^6 = -\dfrac{d^3}{c^3}$

comparing (3) and (4), we get

$\dfrac{b^2}{a^2} = -\dfrac{d^3}{c^3}$

$b^2c^3 = -a^2d^3$

$b^2c^3 + a^2d^3 = 0$

(vii) $v = u + at$; $S = ut + \dfrac{1}{2}at^2$

Sol: Given eqs. are

$v = u + at$ .... (1)

$S = ut + \dfrac{1}{2}at^2$ .... (2)

From (1)

$at = v - u$

$t = \dfrac{v-u}{a}$

put in (2)

$S = u\,(\dfrac{v-u}{a}) + \dfrac{1}{2}a(\dfrac{v-u}{a})^2$

$S = (\dfrac{v-u}{a})\left[u + \dfrac{1}{2}a(\dfrac{v-u}{a})\right]$

$= (\frac{v-u}{a})\left[u+\frac{v-u}{2}\right]$

$= (\frac{v-u}{a})\left[\frac{2u+v-u}{2}\right]$

$= (\frac{v-u}{a})\left[\frac{v+u}{2}\right]$

$S = (\frac{v^2-u^2}{2a})$

$2aS = (v^2-u^2)$

Q.5 Eliminate u from the following equations.

(i) $v = u + at$; $S = ut+\frac{1}{2}at^2$

Sol: Given eqs. are

$v = u + at$ .... (1)

$S = ut+\frac{1}{2}at^2$ .... (2)

from (1)

$u = v - at$

put in (2)

$S = (v - at)\,t + \frac{1}{2}at^2$

$= t\,(v - at + \frac{1}{2}at)$

$= t\left(\frac{2v-2at+at}{2}\right)$

$= t\frac{(2v-2at+at)}{2}$

$S = t\,(v - \frac{1}{2}at)$

(ii) $v = u - gt$; $S = ut + \frac{1}{2}gt^2$

Sol: Given eqs. are

$v = u - gt$ .... (1)

$S = ut + \frac{1}{2}gt^2$ .... (2)

from (1)

$u = v + gt$

put in (2)

$S = (v+gt)\,t + \frac{1}{2}gt^2$

$= t\,(v+gt+\frac{1}{2}gt)$

$= t\left(\frac{2v+2gt+gt}{2}\right)$

$= t\frac{(2v+3gt)}{2}$

$S = t\,(v + \frac{3}{2}gt)$

(iii) $v = u + at$; $2aS = v^2-u^2$

Sol: Given eqs. are

$v = u + at$ .... (1)

$2a\,S = v^2-u^2$ .... (2)

from (1)

$u = v - at$

put in (2)

$2aS = v^2 - (v - at)^2$

$= v^2-v^2-a^2t^2+2atv$

$= 2atv-a^2t^2$

$2aS = at(2v-at)$

$2S = t(2v-at)$

Exercise 2.2

Q.1 Eliminate x from the following equations.

(i) $x-\frac{1}{x} = m$; $x^2+\frac{1}{x^2}=n^2$

Sol: Given eqs. are

$x-\frac{1}{x} = m$ .... (1)

$x^2+\frac{1}{x^2} = n^2$ .... (2)

squaring eq. (1)

$$(x-\frac{1}{x})^2 = m^2$$

$$x^2+\frac{1}{x^2}-2= m^2$$

$$x^2+\frac{1}{x^2} = m^2+2 \qquad \text{......... (3)}$$

comparing (2) and (3), we get

$$n^2 = m^2+2$$

$$n^2-m^2 = 2$$

(ii) $x-\frac{1}{x}=\frac{a}{2}$; $x^2+\frac{1}{x^2}=b^2$

Sol: Given eqs. are

$$x-\frac{1}{x}=\frac{a}{2} \qquad \text{......... (1)}$$

$$x^2+\frac{1}{x^2}=b^2 \qquad \text{......... (2)}$$

squaring both sides of (1)

$$(x-\frac{1}{x})^2 = \left(\frac{a}{2}\right)^2$$

$$x^2+\frac{1}{x^2}-2 = \frac{a^2}{4}$$

$$x^2+\frac{1}{x^2} = \frac{a^2}{4}+2 \qquad \text{......... (3)}$$

comparing (2) and (3), we get

$$\frac{a^2}{4}+2 = b^2$$

$$a^2+8 = 4b^2$$

(iii) $\frac{x^2}{l^2}+\frac{l^2}{x^2}=b^2$; $\frac{l}{x}-\frac{x}{l}=a$

Sol: Given eqs. are

$$\frac{x^2}{l^2}+\frac{l^2}{x^2}=b^2 \qquad \text{......... (1)}$$

$$\frac{l}{x}-\frac{x}{l}=a \qquad \text{......... (2)}$$

squaring both sides of (2)

$$\frac{l^2}{x^2}+\frac{x^2}{l^2}-2=a^2$$

$$\frac{l^2}{x^2}+\frac{x^2}{l^2}=a^2+2 \qquad \text{......... (3)}$$

comparing (1) and (3), we get

$$a^2+2=b^2$$

$$b^2 = a^2+2$$

$$b^2-a^2=2$$

(iv) $\frac{x}{c}+\frac{c}{x}=2a$; $\frac{x}{c}-\frac{c}{x}=3b$

Sol: Given eqs. are

$$\frac{x}{c}+\frac{c}{x}=2a \qquad \text{......... (1)}$$

$$\frac{x}{c}-\frac{c}{x}=3b \qquad \text{......... (2)}$$

squaring both sides of (1)

$$\frac{x^2}{c^2}+\frac{c^2}{x^2}+2=4a^2$$

$$\frac{x^2}{c^2}+\frac{c^2}{x^2}=4a^2-2 \qquad \text{......... (3)}$$

squaring both sides of (2)

$$\frac{x^2}{c^2}+\frac{c^2}{x^2}-2=9b^2$$

$$\frac{x^2}{c^2}+\frac{c^2}{x^2}=9b^2+2 \qquad \text{......... (4)}$$

comparing (3) and (4), we get

$$4a^2-2=9b^2+2$$

$$4a^2-9b^2=4$$

(v) $x+\frac{1}{x}=l$; $x^3+\frac{1}{x^3}=m^3$

Sol: Given eqs. are

$$x+\frac{1}{x}=l \qquad \text{......... (1)}$$

$x^3+\frac{1}{x^3}=m^3$ .... (2)

cubing both side of (1)

$(x+\frac{1}{x})^3=l^3$

$x^3+\frac{1}{x^3}+3(x)(\frac{1}{x})(x+\frac{1}{x})=l^3$

$x^3+\frac{1}{x^3}+3l=l^3$

$x^3+\frac{1}{x^3}=l^3-3l$ ....(3)

from (2) and (3)

$m^3=l^3-3l$

$l^3-3l-m^3=0$

(vi) $x-\frac{1}{x}=p$; $x^2-\frac{1}{x^2}=2q^2$

Sol: Given eqs. are

$x-\frac{1}{x}=p$ .... (1)

$x^2-\frac{1}{x^2}=2q^2$ .... (2)

squaring both sides of (1)

$x^2+\frac{1}{x^2}-2=p^2$

$x^2+\frac{1}{x^2}=p^2+2$

Again squaring both sides

$(x^2+\frac{1}{x^2})^2=(p^2+2)^2$

$x^4+\frac{1}{x^4}+2=p^4+4+4p^2$

$x^4+\frac{1}{x^4}=p^4+4+4p^2-2$

$x^4+\frac{1}{x^4}=p^4+4p^2+2$ ...... (3)

Now squaring both sides of (2)

$(x^2-\frac{1}{x^2})^2=(2q^2)^2$

$x^4+\frac{1}{x^4}-2=4q^4$

$x^4+\frac{1}{x^4}=4q^4+2$ .... (3)

comparing (3) and (4), we get

$p^4+4p^2+2=4q^4+2$

$4q^4+2=p^4+4p^2+2$

$4q^4=p^4+4p^2$

$4q^4=p^2(p^2+4)$

(vii) $x^2-\frac{1}{x^2}=3m^2$; $x^4+\frac{1}{x^4}=n^4$

Sol: Given eqs. are

$x^2-\frac{1}{x^2}=3m^2$ .... (1)

$x^4+\frac{1}{x^4}=n^4$ .... (2)

squaring both sides of (1)

$x^4+\frac{1}{x^4}-2=9m^4$

$x^4+\frac{1}{x^4}=9m^4+2$ .... (3)

comparing (2) and (3), we get

$n^4=9m^4+2$

$n^4-9m^4=2$

(viii) $x-\frac{1}{x}=a$; $x^4+\frac{1}{x^4}=a^4$

Sol: Given eqs. are

$x-\frac{1}{x}=a$ .... (1)

$x^4+\frac{1}{x^4}=a^4$ .... (2)

squaring both sides of (1)

$$x^2+\frac{1}{x^2}-2=a^2$$

$$x^2+\frac{1}{x^2}=a^2+2$$

again squaring both sides

$$x^4+\frac{1}{x^4}+2=a^4+4a^2+4$$

$$x^4+\frac{1}{x^4}=a^4+4a^2+2 \qquad \ldots\ldots (3)$$

comparing (2) and (3), we get

$$a^4=a^4+4a^2+2$$

$$0=4a^2+2$$

$$2a^2+1=0$$

$$2a^2=-1$$

$$a^2=-\frac{1}{2}$$

(ix) $x-\frac{1}{x}=2a$; $x^3-\frac{1}{x^3}=b^3$

Sol: Given eqs. are

$$x-\frac{1}{x}=2a \qquad \ldots\ldots\ldots (1)$$

$$x^3-\frac{1}{x^3}=b^3 \qquad \ldots\ldots\ldots (2)$$

cubing both sides of 1

$$(x-\frac{1}{x})^3=(2a)^3$$

$$x^3-\frac{1}{x^3}-3x\times\frac{1}{x}(x-\frac{1}{x})=8a^3$$

$$x^3-\frac{1}{x^3}-3(2a)=8a^3$$

$$x^3-\frac{1}{x^3}-6a=8a^3$$

$$x^3-\frac{1}{x^3}=8a^3+6a \qquad \ldots\ldots\ldots (3)$$

from (2) & (3), we get

$$b^3=8a^3+6a$$

$$b^3-6a=8a^3$$

$$8a^3=b^3-6a$$

$$8a^3-b^3+6a=0$$

(x) $x^2+\frac{1}{x^2}=a^2$; $x^3+\frac{1}{x^3}=b^3$

Sol: Given eqs. are

$$x^2+\frac{1}{x^2}=a^2 \qquad \ldots\ldots\ldots (1)$$

$$x^3+\frac{1}{x^3}=b^3 \qquad \ldots\ldots\ldots (2)$$

cubing both sides of (1)

$$(x^2+\frac{1}{x^2})^3=(a^2)^3$$

$$x^6+\frac{1}{x^6}+3x\times\frac{1}{x}(x^2+\frac{1}{x^2})=a^6$$

$$x^6+\frac{1}{x^6}+3a^2=a^6$$

$$x^6+\frac{1}{x^6}=a^6-3a^2 \qquad \ldots\ldots\ldots (3)$$

squaring both sides of (2)

$$x^6+\frac{1}{x^6}+2=b^6$$

$$x^6+\frac{1}{x^6}=b^6-2 \qquad \ldots\ldots\ldots (4)$$

comparing (3) & (4), we get

$$a^6-3a^2=b^6-2$$

$$b^6-2=a^6-3a^2$$

$$b^6-2-a^6+3a^2=0$$

$$b^6-a^6+3a^2=2$$

(xi) $\sqrt{x}+\frac{1}{\sqrt{x}}=\sqrt{a}$; $x-\frac{1}{x}=b$

Sol: Given eqs. are

$$\sqrt{x}+\frac{1}{\sqrt{x}}=\sqrt{a} \qquad \text{.........} (1)$$

$$x-\frac{1}{x}=b \qquad \text{.........} (2)$$

squaring both sides of (1)

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=\left(\sqrt{a}\right)^2$$

$$x+\frac{1}{x}+2=a$$

$$x+\frac{1}{x}=a-2$$

Again squaring both sides

$$(x+\frac{1}{x})^2=(a-2)^2$$

$$x^2+\frac{1}{x^2}+2=a^2-4a+4$$

$$x^2+\frac{1}{x^2}=a^2-4a+2 \qquad \text{......} (3)$$

Again squaring both sides of (2)

$$\left(x-\frac{1}{x}\right)^2=(b)^2$$

$$x^2+\frac{1}{x^2}-2=b^2$$

$$x^2+\frac{1}{x^2}=b^2+2 \qquad \text{.........} (4)$$

comparing (3) and (4), we get

$$a^2-4a+2=b^2+2$$

$$a^2-4a=b^2$$

(xii) $ax+\frac{1}{ax}=3p$; $a^2x^2-\frac{1}{a^2x^2}=4l^2$

Sol: Given eqs. are

$$ax+\frac{1}{ax}=3p \qquad \text{.........} (1)$$

$$a^2x^2-\frac{1}{a^2x^2}=4l^2 \qquad \text{.........} (2)$$

squaring both sides of 1

$$a^2x^2+\frac{1}{a^2x^2}+2=9p^2$$

$$a^2x^2+\frac{1}{a^2x^2}=9p^2-2$$

Again squaring both sides

$$a^4x^4+\frac{1}{a^4x^4}+2=81p^4-36p^2+4$$

$$a^4x^4+\frac{1}{a^4x^4}=81p^4-36p^2+2 \quad \text{...} (3)$$

squaring both sides of (2)

$$a^4x^4+\frac{1}{a^4x^4}-2=16l^4$$

$$a^4x^4+\frac{1}{a^4x^4}=16l^4+2 \qquad \text{......} (4)$$

comparing (3) and (4), we get

$$81p^4-36p^2+2=16l^4+2$$

$$81p^4-36p^2-16l^4=0$$

(xiii) $\frac{x^3}{a^3}+\frac{a^3}{x^3}=m$; $\frac{x^3}{a^3}-\frac{a^3}{x^3}=n$

Sol: Given eqs. are

$$\frac{x^3}{a^3}+\frac{a^3}{x^3}=m \qquad \text{.........} (1)$$

$$\frac{x^3}{a^3}-\frac{a^3}{x^3}=n \qquad \text{.........} (2)$$

squaring both sides of (1)

$$\frac{x^6}{a^6}+\frac{a^6}{x^6}+2=m^2$$

$$\frac{x^6}{a^6}+\frac{a^6}{x^6}=m^2-2 \qquad \text{.........} (3)$$

now squaring both sides of (2)

$$\frac{x^6}{a^6}+\frac{a^6}{x^6}-2=n^2$$

$$\frac{x^6}{a^6}+\frac{a^6}{x^6}=n^2+2 \qquad \text{.........} (4)$$

comparing (3) and (4), we get

$$m^2 - 2 = n^2 + 2$$

$$m^2 - n^2 = 4$$

Q.2 Eliminate t from the following equations.

(i) $a(1+t^2) = 2xt$; $b(1-t^2) = 2yt$

Sol: Given eqs. are

$$a(1+t^2) = 2xt \quad \text{.........} (1)$$

$$b(1-t^2) = 2yt \quad \text{.........} (2)$$

from (1)

$$\frac{1+t^2}{2t} = \frac{x}{a}$$

squaring both sides

$$\frac{x^2}{a^2} = \frac{(1+t^2)^2}{4t^2} \quad \text{.........} (3)$$

from (2)

$$\frac{1-t^2}{2t} = \frac{y}{b}$$

squaring both sides

$$\frac{y^2}{b^2} = \frac{(1-t^2)^2}{4t^2} \quad \text{.........} (4)$$

subtracting (4) from (3), we get

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{(1+t^2)^2}{4t^2} - \frac{(1-t^2)^2}{4t^2}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{(1+t^2)^2 - (1-t^2)^2}{4t^2}$$

$$= \frac{1+t^4+2t^2-1-t^4+2t^2}{4t^2}$$

$$= \frac{4t^2}{4t^2}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

ii) $y = \dfrac{2t}{b(1-t^2)}$; $\dfrac{x}{a} = \dfrac{1+t^2}{1-t^2}$

Sol: Given eqs. are

$$yb = \frac{2t}{(1-t^2)} \quad \text{.........} (1)$$

$$\frac{x}{a} = \frac{1+t^2}{1-t^2} \quad \text{.........} (2)$$

from (1) squaring both sides.

$$b^2y^2 = \frac{4t^2}{(1-t^2)^2} \quad \text{.........} (3)$$

from (2) squaring both sides.

$$\frac{x^2}{a^2} = \frac{(1+t^2)^2}{(1-t^2)^2} \quad \text{.........} (4)$$

subtracting (4) from (3)

$$y^2b^2 - \frac{x^2}{a^2} = \frac{4t^2}{1+t^4-2t^2} - \frac{1+t^4+2t^2}{1+t^4-2t^2}$$

$$= \frac{4t^2-1-t^4-2t^2}{t^4-2t^2+1}$$

$$= \frac{-\cancel{(t^4-2t^2+1)}}{\cancel{(t^4-2t^2+1)}}$$

$$= -1$$

$$y^2b^2 - \frac{x^2}{a^2} = -1$$

(iii) $x = \dfrac{1+t^2}{2at}$; $y = \dfrac{1-t^2}{2bt}$

Sol: Given eqs. are

$$x = \frac{1+t^2}{2at} \Rightarrow ax = \frac{1+t^2}{2t} \quad \text{.........} (1)$$

$$y = \frac{1-t^2}{2bt} \Rightarrow by = \frac{1-t^2}{2t} \quad \text{.........} (2)$$

from (1) squaring both sides.

$$a^2x^2 = \frac{(1+t^2)^2}{4t^2} \quad \text{.........} (3)$$

from (2) squaring both sides.

$$b^2y^2 = \frac{(1-t^2)^2}{4t^2} \quad \text{.........} (4)$$

subtracting (4) from (3)

$$a^2x^2 - b^2y^2 = \frac{1+t^4+2t^2}{4t^2} - \frac{1+t^4-2t^2}{4t^2}$$

$$= \frac{1+t^4+2t^2-1-t^4+2t^2}{4t^2}$$

$$= \frac{4t^2}{4t^2}$$

$$a^2x^2 - b^2y^2 = 1$$

(iv) $\frac{x}{a} = \frac{1-t^2}{1+t^2}$; $\frac{y}{b} = \frac{2t}{1+t^2}$

Sol: Given eqs. are

$$\frac{x}{a} = \frac{1-t^2}{1+t^2} \quad \text{.........} (1)$$

$$\frac{y}{b} = \frac{2t}{1+t^2} \quad \text{.........} (2)$$

from (1) squaring both sides.

$$\frac{x^2}{a^2} = \frac{(1-t^2)^2}{(1+t^2)^2} \quad \text{.........} (3)$$

from (2) squaring both sides.

$$\frac{y^2}{b^2} = \frac{4t^2}{(1+t^2)^2} \quad \text{.........} (4)$$

adding (3) and (4)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1-2t^2+t^4}{1+2t^2+t^4} + \frac{4t^2}{1+2t^2+t^4}$$

$$= \frac{1-2t^2+t^4+4t^2}{1+2t^2+t^4}$$

$$= \frac{\cancel{1+2t^2+t^4}}{\cancel{1+2t^2+t^4}}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Exercise 2.3

Find the relation independent of x for the following equations.

Q.1

Sol: Given eqs are

$$ax^2 - 4x + 5 = 0 \quad \text{.........} (1)$$

$$bx^2 + cx - 6 = 0 \quad \text{.........} (2)$$

from (1) and (2)

$$\begin{array}{cccccc} a & -4 & 5 & a & -4 & 5 \\ b & c & -6 & b & c & -6 \end{array}$$

$$\frac{x^2}{24-5c} = \frac{x}{5b+6a} = \frac{1}{ac+4b}$$

Now $x^2 = \frac{24-5c}{ac+4b}$ .... (3)

Also $\frac{x}{5b+6a} = \frac{1}{ac+4b}$

$$x = \frac{5b+6a}{ac+4b}$$

squaring both sides

$$x^2 = \frac{(5b+6a)^2}{(ac+4b)^2} \quad \text{.........} (4)$$

comparing (3) and (4), we get

$$\frac{24-5c}{ac+4b} = \frac{(5b+6a)^2}{(ac+4b)^2}$$

$$24-5c = \frac{(5b+6a)^2}{(ac+4b)^2} \times (ac+4b)$$

$$\Rightarrow \quad (5b+6a)^2 = (24-5c)(ac+4b)$$

Q.2 $2x^2 - x + p = 0$; $x^2 - 3x - q = 0$

Sol: Given eqs. are

$$2x^2 - x + p = 0 \quad \text{.........} (1)$$

$$x^2 - 3x - q = 0 \quad \text{.........} (2)$$

from (1) and (2)

$$\begin{array}{cccccc} 2 & -1 & p & 2 & -1 & p \\ 1 & -3 & -q & 1 & -3 & -q \end{array}$$

$$\frac{x^2}{q+3p} = \frac{x}{p+2q} = \frac{1}{-6+1}$$

$$\frac{x^2}{q+3p} = \frac{x}{p+2q} = -\frac{1}{5}$$

now $\dfrac{x^2}{q+3p} = -\dfrac{1}{5}$

$\Rightarrow \quad x^2 = -\dfrac{q+3p}{5}$ .... (3)

also $\dfrac{x}{p+2q} = \dfrac{-1}{5}$

$$x = \frac{-(p+2q)}{5}$$

squaring both sides

$x^2 = \dfrac{(p+2q)^2}{25}$ .... (4)

comparing (3) and (4)

$$\frac{(p+2q)^2}{25} = -\frac{q+3p}{5}$$

$\Rightarrow \quad (p+2q)^2 = -5\,(q+3p)$

Q.3 $x^2-4x+l=0$; $2x^2-x-m=0$

Sol: Given eqs. are

$x^2-4x+l=0$ .... (1)

$2x^2-x-m=0$ .... (2)

$$\begin{array}{cccccc} 1 & -4 & l & 1 & -4 & l \\ 2 & -1 & -m & 2 & -1 & -m \end{array}$$

$$\frac{x^2}{4m+l} = \frac{x}{2l+m} = \frac{1}{-1+8}$$

$$\frac{x^2}{4m+l} = \frac{x}{2l+m} = \frac{1}{7}$$

now $\dfrac{x^2}{4m+l} = \dfrac{1}{7}$

$\Rightarrow \quad x^2 = \dfrac{4m+l}{7}$ .... (3)

also $x = \dfrac{2l+m}{7}$

squaring both sides

$x^2 = \dfrac{(2l+m)^2}{49}$ .... (4)

comparing (3) and (4)

$$\frac{(2l+m)^2}{49} = \frac{4m+l}{7}$$

$\Rightarrow \quad (2l+m)^2 = 7(4m+l)$

Q.4 $x^2-2x+l=0$; $-x^2+3x+m=0$

Sol: Given eqs. are

$x^2-2x+l=0$ .... (1)

$-x^2+3x+m=0$ .... (2)

from (1) and (2)

$$\begin{array}{cccccc} 1 & -2 & l & 1 & -2 & l \\ -1 & 3 & m & -1 & 3 & m \end{array}$$

$$\frac{x^2}{-2m-3l} = \frac{x}{-l-m} = \frac{1}{3-2}$$

$$\frac{x^2}{-(2m+3l)} = \frac{x}{-(l+m)} = 1$$

Now $\dfrac{x^2}{-(2m+3l)} = 1$

$\Rightarrow \quad x^2 = -(2m+3l)$ .... (3)

also $\dfrac{x}{-(l+m)} = \dfrac{1}{1}$

$$x = -(l+m)$$

squaring both sides

$x^2 = (l+m)^2$ .... (4)

comparing (3) and (4)

$(l+m)^2 = -(2m+3l)$

Q.5 $-2x^2-2x = p$; $-x^2+x = q$

Sol: Given eqs. are

$$-2x^2-2x-p=0 \quad \text{.........} (1)$$

$$-x^2+x-q=0 \quad \text{.........} (2)$$

from (1) and (2)

$$\begin{array}{cccccc} -2 & -2 & p & -2 & -2 & -p \\ -1 & 1 & q & -1 & 1 & -q \end{array}$$

$$\frac{x^2}{2q+p}=\frac{x}{p-2q}=\frac{1}{-2-2}$$

$$\frac{x^2}{2q+p}=\frac{x}{p-2q}=-\frac{1}{4}$$

$$\frac{x^2}{2q+p}=-\frac{1}{4}$$

$$\Rightarrow \quad x^2=-\frac{2q+p}{4} \quad \text{.........} (3)$$

also $\frac{x}{p-2q}=-\frac{1}{4}$

$$x=-\frac{(p-2q)}{4}$$

squaring both sides

$$x^2=\frac{(p-2q)^2}{16} \quad \text{.........} (4)$$

comparing (3) and (4), we get

$$\frac{(p-2q)^2}{16}=-\frac{2q+p}{4}$$

$$(p-2q)^2=-4(2q+p)$$

Q.6 $5x^2-3x+p=0$; $6x^2-5x-q=0$

Sol: Given eqs. are

$$5x^2-3x+p=0 \quad \text{.........} (1)$$

$$6x^2-5x-q=0 \quad \text{.........} (2)$$

from (1) and (2)

$$\begin{array}{cccccc} 5 & -3 & p & 5 & -3 & p \\ 6 & -5 & -q & 6 & -5 & -q \end{array}$$

$$\frac{x^2}{3q+5p}=\frac{x}{6p+5q}=\frac{1}{-25+18}$$

Now $\frac{x^2}{3q+5p}=\frac{x}{6p+5q}=\frac{-1}{7}$

$$\Rightarrow \quad x^2=-\frac{3q+5p}{7} \quad \text{.........} (3)$$

also $\frac{x}{6p+5q}=\frac{-1}{7}$

$$x=\frac{-(6p+5q)}{7}$$

squaring both sides

$$x^2=\frac{(6p+5q)^2}{49} \quad \text{.........} (4)$$

comparing (3) and (4), we gt

$$\frac{(6p+5q)^2}{49}=\frac{-(3q+5p)}{7}$$

$$\Rightarrow \quad (6p+5q)^2=-7(5p+3q)$$

Q.7 $mx^2+3x+2=0$; $nx^2+5x+1=0$

Sol: Given eqs. are

$$mx^2+3x+2=0 \quad \text{.........} (1)$$

$$nx^2+5x+1=0 \quad \text{.........} (2)$$

from (1) and (2)

$$\begin{array}{cccccc} m & 3 & 2 & m & 3 & 2 \\ n & 5 & 1 & n & 5 & 1 \end{array}$$

$$\frac{x^2}{3-10}=\frac{x}{2n-m}=\frac{1}{5m-3n}$$

$$\frac{x^2}{-7}=\frac{x}{2n-m}=\frac{1}{5m-3n}$$

now $\frac{x^2}{-7}=\frac{1}{5m-3n}$

$$\Rightarrow \quad x^2=-\frac{7}{5m-3n} \quad \text{.........} (3)$$

also $\frac{x}{2n-m}=\frac{1}{5m-3n}$

$$x=\frac{2n-m}{5m-3n}$$

squaring both sides

$$x^2 = \frac{(2n-m)^2}{(5m-3n)^2} \quad \cdots\cdots\cdots (4)$$

comparing (3) and (4), we get

$$\frac{(2n-m)^2}{(5m-3n)^2} = -\frac{7}{5m-3n}$$

$$\Rightarrow \quad (2n-m)^2 = -7\,(5m-3n)$$

Q.8 $7x^2 - sx + 5 = 0\ ;\ 3x^2 + tx - 6 = 0$

Sol: Given eqs. are

$$7x^2 - sx + 5 = 0 \quad \cdots\cdots\cdots (1)$$

$$3x^2 + tx - 6 = 0 \quad \cdots\cdots\cdots (2)$$

from (1) and (2)

$$\begin{array}{cccccc} 7 & -s & 5 & 7 & -s & 5 \\ 3 & t & -6 & 3 & t & -6 \end{array}$$

$$\frac{x^2}{6s-5t} = \frac{x}{15+42} = \frac{1}{7t+3s}$$

$$\frac{x^2}{6s-5t} = \frac{x}{57} = \frac{1}{7t+3s}$$

now $\quad \dfrac{x^2}{6s-5t} = \dfrac{1}{7t+3s}$

$$\Rightarrow \quad x^2 = \frac{6s-5t}{7t+3s} \quad \cdots\cdots\cdots (3)$$

also $\quad \dfrac{x}{57} = \dfrac{1}{7t+3s}$

$$x = \frac{57}{7t+3s}$$

squaring both sides

$$x^2 = \frac{(57)^2}{(7t+3s)^2} \quad \cdots\cdots\cdots (4)$$

comparing (3) and (4), we get

$$\frac{(57)^2}{(7t+3s)^2} = \frac{6s-5t}{7t+3s}$$

$$\Rightarrow \quad (57)^2 = (6s-5t)\,(7t+3s)$$

EXAMPLES

Example 1: Eliminate x from the following equations where $p \neq 0$, $r \neq 0$:

$px - q = 0 \quad$ (i)

$rx - s = 0 \quad$ (ii)

Solution: **By substituting method**

Equation (i) gives

$px = q$

or $\quad x = \dfrac{q}{p}$

Putting the value of x in equation (ii), we get

$$r.\frac{q}{p} - s = 0$$

or $\quad rq - ps = 0$

or $\quad qr = ps$

This equation is independent of x so it is eliminant.

By comparison: Equation (i) give

$px = q$

or $\quad x = \dfrac{q}{p} \quad$ (iii)

Equation (ii) give

$rx = s$

or $\quad x = \dfrac{s}{r} \quad$ (iv)

Comparing equation (iii) and (iv), we get

$$\frac{q}{p} = \frac{s}{r}$$

$$ps = qr$$

This equation is independent of x so is eliminant.

Example 2: Eliminate x from $lx^2+mx+n=0$ *and* $ax+b=0$ by substitution $l \neq 0$, $a \neq 0$

Solution: $ax + b = 0$ (i)

$lx^2 + mx + n = 0$ (ii)

equation (i) gives

$ax = -b$

or $x = -\dfrac{b}{a}$

Putting the value of x in equation (ii), we get

$$l\left(\frac{-b}{a}\right)^2 + m\left(\frac{-b}{a}\right) + n = 0$$

or $l\left(\dfrac{b^2}{a^2}\right) - m\dfrac{b}{a} + n = 0$

or $lb^2 - mab + na^2 = 0$ (iii)

Therefore, equation (iii) is the eliminant.

Example 3: Find a relation independent of *t* from the following equations:

$ct^5 = d$, $at^3 = b$, $c \neq 0$, $a \neq 0$

Solution: $ct^5 = d$ (i)

$at^3 = b$ (ii)

equation (ii) given

$at^3 = b$

$t^3 = \dfrac{b}{a}$

Taking 5^{th} power of both sides

$$\left(t^3\right)^5 = \left(\frac{b}{a}\right)^5$$

$$t^{15} = \frac{b^5}{a^5} \text{........(iii)}$$

From (i)

$ct^5 = d$

$t^5 = \dfrac{d}{c}$

Taking cube on both sides

$$\left(t^5\right)^3 = \left(\frac{d}{c}\right)^3$$

$$t^{15} = \frac{d^3}{c^3} \text{..........(iv)}$$

comparing (iii) and (iv)

$$\frac{b^5}{a^5} = \frac{d^3}{c^3}$$

$$a^5d^3 = c^3b^5$$

It is the required relation which is independent of t.

Example 4: Eliminate x from the following equations by using the formulae.

$$x+\frac{1}{x}=l \; ; \; x^2+\frac{1}{x^2}=m^2$$

Solution:

$x+\dfrac{1}{x}=l$ .... (i)

$x^2+\dfrac{1}{x^2}=m^2$ .... (ii)

Taking square of both sides of the equation (i), we have

$$\left(x+\frac{1}{x}\right)^2 = (l)^2$$

or $x^2+\dfrac{1}{x^2}+2=l^2$

or $x^2+\dfrac{1}{x^2}=l^2-2$ .... (iii)

Comparing equations (ii) and (iii), we get

$l^2-2=m^2$

This is the required relation

Example 5: Eliminate t from the following equations.

$$x = \frac{2at}{1+t^2} \qquad \text{(i)}$$

$$y = \frac{b(1-t^2)}{1+t^2} \qquad \text{(ii)}$$

Solution:

Equation (i) gives

$$\frac{x}{a} = \frac{2t}{1+t^2}$$

$$\left(\frac{x}{a}\right)^2 = \left(\frac{2t}{1+t^2}\right)^2$$

(Taking square of both sides)

or $$\frac{x^2}{a^2} = \frac{4t^2}{1+2t^2+t^4} \quad \ldots\ldots\text{(iii)}$$

Equation (ii) gives

$$\frac{y}{b} = \frac{1-t^2}{1+t^2}$$

or $$\left(\frac{y}{b}\right)^2 = \left(\frac{1-t^2}{1+t^2}\right)^2$$

(Taking square of both sides)

or $$\frac{y^2}{b^2} = \frac{1-2t^2+t^4}{1+2t^2+t^4} \quad \ldots\ldots\text{(iv)}$$

From equations (iii) and (iv), we have

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{4t^2}{1+2t^2+t^4} + \frac{1-2t^2+t^4}{1+2t^2+t^4}$$

$$= \frac{4t^2+1-2t^2+t^4}{1+2t^2+t^4}$$

$$= \frac{\cancel{1+2t^2+t^4}}{\cancel{1+2t^2+t^4}}$$

$$= 1$$

Thus $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Is the required relation.

Example 6: Eliminate x from the following equations where $b \neq 0$, $d \neq 0$:

$$a = b\left(x^2 + \frac{1}{x^2}\right) \qquad \text{(i)}$$

$$c = d\left(x^3 + \frac{1}{x^3}\right) \qquad \text{(ii)}$$

Solution: Dividing both the sides of equation (i) by b, we have

$$\frac{a}{b} = x^2 + \frac{1}{x^2}$$

Taking cube of both sides, we have

$$\left(\frac{a}{b}\right)^3 = \left(x^2 + \frac{1}{x^2}\right)^3$$

or $$\frac{a^3}{b^3} = x^6 + \frac{1}{x^6} + 3\left(x^2 + \frac{1}{x^2}\right)$$

or $$\frac{a^3}{b^3} = x^6 + \frac{1}{x^6} + 3\left(\frac{a}{b}\right)$$

or $$x^6 + \frac{1}{x^6} = \frac{a^3}{b^3} - \frac{3a}{b} \quad \ldots\ldots\text{(iii)}$$

Dividing both sides of equation (ii) by d, we have

$$\frac{c}{d} = x^3 + \frac{1}{x^3}$$

or $$\left(\frac{c}{d}\right)^2 = \left(x^3 + \frac{1}{x^3}\right)^2$$

(Taking square of both sides)

or $$\frac{c^2}{d^2} = x^6 + \frac{1}{x^6} + 2$$

or $$x^6 + \frac{1}{x^6} + 2 = \frac{c^2}{d^2}$$

or $$x^6 + \frac{1}{x^6} = \frac{c^2}{d^2} - 2 \ldots\ldots$$

Comparing equations (iii) and (iv), we get

$$\frac{a^3}{b^3}-\frac{3a}{b}=\frac{c^2}{d^2}-2$$

or $$\frac{a^3-3ab^2}{b^3}=\frac{c^3-2d^2}{d^2}$$

or $$\left(a^3-3ab^2\right)d^2=\left(c^2-2d^2\right)b^3$$

Thus

$$\left(a^3-3ab^2\right)d^2=\left(c^2-2d^2\right)b^3$$

is the required relation

Example 7: Eliminate x by using cross multiplication method from the following quadratic equations.

$ax^2+bx+c=0$, $lx^2+mx+n=0$

Solution: $ax^2+bx+c=0$..........(i)

$lx^2+mx+n=0$.........(ii)

$$\begin{array}{cccccc} a & b & c & a & b & c \\ l & m & n & l & m & n \end{array}$$

$$\frac{x^2}{bn-mc}=\frac{x}{cl-an}=\frac{1}{am-bl}\text{.......(iii)}$$

From equation (iii), we have

$$\frac{x^2}{bn-mc}=\frac{1}{am-bl}\Rightarrow x^2=\frac{bn-mc}{am-bl}\text{....(iv)}$$

$$\frac{x}{cl-an}=\frac{1}{am-bl}\Rightarrow x=\frac{cl-an}{am-bl}\text{....(v)}$$

Taking square of both sides of equation (v), we get

$$x^2=\frac{(cl-an)^2}{(am-bl)^2}\text{..............(vi)}$$

From equations (iv) and (vi), we get

$$\frac{bn-cm}{am-bl}=\frac{(cl-an)^2}{(am-bl)^2}$$

$$(am-bl)(bn-cm)=(cl-an)^2$$

Objective

Q.1 Four answers of each item are given from which only one is true. Tick the correct answer.

1. Eliminating 'x' from $x^2+\frac{1}{x^2}=m^2$ and $x+\frac{1}{x}=n$ we get.
   (Lahore Board, 2008, 2010)
   (a) $m^2-n^2=2$ (b) $m^2+n^2=2$
   (c) $m^2-n^2=-2$ (d) $m^2+n^2=-2$
2. At least _______ equations are required for elimination of one variable
   (a) 2 (b) 3
   (c) 4 (d) 1
3. To eliminate any variable from the equations is called _________.
   (a) Eliminate (b) Elimination
   (c) Solution set (d) None of these
4. If 2x = a and 3x = b, then the relation free from x is ___________.
   (a) 2a = 3b (b) 3a = 2b
   (c) a =b (d) None of these.
5. Which is free relation from 'x' for equations $x=\frac{1}{3n}$ and x = 2m.
   (Lahore Board 2009, 2010)
   (a) 2mn = 1 (b) m = 3n

(c) $2m = n$ (d) $6mn = 1$

6. Which is free relation from 'x' for equations $px - q = 0$ and $rx - s = 0$
(a) $rp = ps$ (b) $rq = ps$
(c) $sq = pr$ (d) $r = s$

7. Which is free relation from 'x' for equations $x + y = a$ and $x^2 + y^2 = b^2$.
(a) $2y(y - a) = b^2 - a^2$
(b) $2y = b^2 - a^2$
(c) $y - a = b^2 - a^2$ (d) None of these

8. Which is free relation from 'x' for equations
$ax + b = 0$ and $\ell x^2 + mx + n = 0$.
(a) $\ell b^2 - mab + na^2 = 0$
(b) $\ell b^2 - mab$
(c) $\ell b^2 + na^2$
(d) $\ell b^2 + mab - na^2 = 0$

9. Eliminating 'z' from $m - z = 2$ and $n + z = 4$, we get. ***(Lahore Board 2009)***
(a) $m + n = 6$ (b) $m - n = 6$
(c) $m + n = 2$ (d) $m - n = 2$

10. Eliminating 'x' from $x^2 + \frac{1}{x^2} = m^2$ and $x - \frac{1}{x} = \ell$ we get.
(a) $m^2 + \ell^2 = 2$ (b) $\ell^2 = m^2 + 2$
(c) $m^2 - \ell^2 = 2$ (d) $m^2 + \ell^2 = -2$

11. Which is free relation from 't' for equations $at^2 = x$ and $bt^3 = y$
(a) $b^2x^3 = a^3y^2$ (b) $b^3x^2 = a^2y^3$
(c) $b^4x^2 = a^2y^3$ (d) None of these

12. Eliminating 't' from $t = \frac{1}{4q^2}$ and $3p^2 = \frac{1}{t}$, we get.
(Lahore Board 2008)
(a) $3p^2q^2 = 4$ (b) $12p^2q^2 = 1$
(c) $3p^2 = 4q^2$ (d) $4p^2 = 3q^2$

13. Eliminating 'm' from $m^3 = 2x$ and $m^2 = \frac{y}{2}$, we get.
(Lahore Board 2008)
(a) $y^2 = 32x^3$ (b) $y^3 = 32x^2$
(c) $x^2 = 32y^3$ (d) $x^3 = 32y^2$

14. Eliminant shows that the solution set of the equations is ______.
(a) empty (b) non-empty
(c) zero (d) None of these

15. Eliminating 'x' from $ax - b = 0$ and $cx^2 + dx + f = 0$, we get.
(a) $cb^2 + abd + a^2f = 0$
(b) $cb^2 + abd - a^2f = 0$
(c) $cb^2 - abd - a^2f = 0$
(d) None of these

16. Eliminating 't' from $x = \sqrt{2}t$ and $y = \sqrt{6}t$, we get.
(a) $y = \sqrt{3}x$ (b) $y = \sqrt{2}x$
(c) $y^2 = \sqrt{3}x$ (d) None of these

17. Which is free relation from 'x' for equations $x + \frac{1}{x} = \ell$ and $x^2 + \frac{1}{x^2} = m^2$.
(a) $\ell^2 = m^2 - 2$ (b) $\ell^2 - 2 = m^2$
(c) $\ell^2 = m^2$ (d) None of these

18. Which is free relation from 't' for equations $at = x$ and $2at = y$.
(a) $y = 2x$ (b) $2x = \frac{1}{y}$
(c) $x = y$ (d) None of these

19. The relation free from 'x' for equations $x = a$ and $x = \frac{1}{b}$ is ________.

(a) $ab = 1$ (b) $a = b$

(c) $\frac{b}{a}$ (d) $a = 1$

20. The relation independent of 't' from $ct^5 = d$ and $at^3 = b$ is ________.

(a) $a^3 d^5$ (b) $a^5 d^3 = c^3 b^5$

(c) $a^5 b^3 = b^3 c^5$ (d) None of these

21. The relation free from 'y' for equations $y = 3t$ and $yt = 1$ is_______.

(a) $3t^2 = 1$ (b) $3t = 1$

(c) $9t = 1$ (d) $t = \frac{1}{3}$

22. The relation free from 't' for equations $y = 3t$ and $yt = 1$ is________.

(a) $y = 3$ (b) $y^2 = 3$

(c) $y^2 = \frac{1}{3}$ (d) $3y^2 = 1$

23. The eliminant by eliminating 'm' for equations $m + p = 3s$ and $m - q = 2r$ is __________. ***(Lahore Board 2010)***

(a) $p + q = 3s - 2r$

(b) $3s + 2r = p + q$

(c) $3s + 2r = p - q$

(d) $3s - 2r = p - q$

24. The relation free from 'x' for equations $x + \frac{1}{x} = a$ and $x + \frac{1}{x} = b$ is_____.

(a) $a = b$ (b) $\frac{a}{b}$

(c) $ab = 1$ (d) $a^2 = b^2$

25. The relation free from 'x' for equations $x^2 + \frac{1}{x^2} = 4$ and $x - \frac{1}{x} = b$ is___________.

(a) $b^2 = 1$ (b) $b^2 = 2$

(c) $b^2 = 4$ (d) $b^2 = 3$

26. *Eliminate 'x' from $x + \frac{1}{x} = \ell^2$ and* $x^2 + \frac{1}{x^2} = m^2$, we get_______

(a) $\ell^4 - m^2 = -2$ (b) $\ell^4 - m^2 = 2$

(c) $\ell^4 + m^2 = 2$ (d) $\ell^4 + m^2 = -2$

27. The relation free from 'x' for equations $xt = \ell$ and $\frac{x}{m} = t$ is______

(a) $\ell = mt^2$ (b) $\ell = mt$

(c) $t = m\ell$ (d) $t^2 = m\ell$

28. The relation free from 'x' for equations $p(x + a) = q(b + x)$ and $p(c+x) = q(x+d)$ is_________.

(a) $q(b - d) = p(a - c)$

(b) $q(b + b) = p(a + c)$

(c) $q(a - c) = p(b - d)$

(d) None of these

29. Eliminating 'x' from $x - \frac{1}{x} = \frac{a}{2}$ and $x^2 + \frac{1}{x^2} = b^2$, we get_________.

(a) $4b^2 = a^2 + 8$ (b) $4b = a + 8$

(c) $4b^2 = a + 8$ (d) $4b = a^2 + 8$

30. The relation free from 'x' for equations $\frac{x^3}{a^3} + \frac{a^3}{x^3} = m$ and $\frac{x^3}{a^3} - \frac{a^3}{x^3} = n$ is___________.

(a) $m^2 + n^2 = 4$ (b) $m^2 - n^2 = 4$

(c) $m^6 - n^6 = 4$ (d) $m^4 = n^4 = 2$

31. Eliminating 't' form $v = u + at$ and $s = ut + \frac{1}{2}at^2$, we get_______.

(a) $2as = v^2 - u^2$ (b) $2as = u^2 - v^2$
(c) $2s = v^2 - u^2$ (d) $2s = v^2 + u^2$

32. The relation free from 'x' for equation $x - pq = 0$ and $\frac{x}{\ell} = m$ is _____.

(a) $pq = \ell$ (b) $q\ell = qm$
(c) $pq - \ell m = 0$ (d) $pq = \frac{\ell}{m}$

33. The relation free from 't' for equation $at^3 - b = 0$ and $ct^2 + d = 0$ is _____.

(a) $b^2 c^3 + a^2 d^3 = 0$
(b) $b^2 c^3 - a^2 d^3 = 0$
(c) $b^2 c^3 = a^2 d^3$
(d) None of these

34. The relation free from 't' for equation $t^2 = \frac{1}{p}$ and $t^3 = q$ is _____.

(a) $1 = p^3q^2$ (b) $1 = q^3p^2$
(c) $1 = p^2$ (d) $q^3 = 1$

35. Eliminating 'x' from $x - y = 2$ and $2x + 3y = 5$, we get

(a) $5y = 1$ (b) $4y = 1$
(c) $3y = 1$ (d) $y = 1$

Answers

1.	c	**2.**	a	**3.**	b	**4.**	b	**5.**	d	**6.**	b	**7.**	a
8.	a	**9.**	a	**10.**	c	**11.**	a	**12.**	c	**13.**	b	**14.**	b
15.	a	**16.**	a	**17.**	b	**18.**	a	**19.**	a	**20.**	b	**21.**	a
22.	b	**23.**	a	**24.**	a	**25.**	b	**26.**	b	**27.**	a	**28.**	a
29.	a	**30.**	b	**31.**	a	**32.**	c	**33.**	a	**34.**	a	**35.**	a